Pergamon

J. Appl. Maths Mechs, Vol. 62, No. 1, pp. 115–122, 1998

0021–8928/98 \$24.00+0.00

PII: S0021–8928(98)00013-6

SYNTHESIS OF A CONTROL IN A NON-LINEAR DYNAMICAL SYSTEM BASED ON DECOMPOSITION†

S. A. RESHMIN and F. L. CHERNOUS'KO

Moscow

(*Received* 13 *February* 1997)

A non-linear controllable dynamical system with many degrees of freedom, described by Lagrange equations of the second kind, is considered. Geometric constraints are imposed on the magnitudes of the controls. It is assumed that, in the equations of motion, the kinetic energy matrix is close to a certain constant diagonal matrix. It is possible, for example, to reduce the equations of motion of robots, the drives of which have large gear ratios, to a system of this kind. A problem is formulated on the transfer of a system in a finite time from a specified initial state to a final state with zero velocities. The method of decomposition [1] is used to construct the equations. Sufficient conditions are found subject to which the maximum values of the non-linear terms in the equations of motion do not exceed the permissible magnitudes of the controls. In this case, non-linearities are treated as limited perturbations and the system is decomposed into independent, linear, second-order subsystems. A feedback control is specified for these subsystems which guarantees that each of them is brought into the final state for any permissible perturbations. The control has a simple structure. Applications of the proposed approach to problems in the control of manipulating robots are considered.

This paper is related to [1–4] but other conditions for the implementation of the method of decomposition are given here. Another method for controlling dynamical systems, based on decomposition, has also been proposed in [5, 6].

1. FORMULATION OF THE PROBLEM

A non-linear controllable dynamical system with n degrees of freedom is considered. The motion of this system is described by the Lagrange equations

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}=U_i+Q_i \tag{1.1}$$

Here, $q=(q_1,\ldots,q_n)$ is a vector of the generalized coordinates, $q\in D\subset R^n$, U_i are the generalized controlling forces to be determined, Q_i are the other generalized forces and T is the kinetic energy of the system, which is given by the quadratic form

$$T(q,\dot{q})=\frac{1}{2}(A(q)\dot{q},\dot{q})=\frac{1}{2}\sum_{j,k}A_{jk}(q)\dot{q}_j\dot{q}_k \tag{1.2}$$

where $A(q)$ is a symmetric positive-definite matrix with elements $A_{jk}(q)$. Above and everywhere henceforth, the subscripts i, j and k take the values $1, 2, \ldots, n$.

The domain D, in which the motions of the system being considered can occur, is specified in the form of independent constraints on the coordinates q_i

$$D=\{q:\ q_i^-\leqslant q_i\leqslant q_i^+\} \tag{1.3}$$

Constraints are also imposed on the generalized control forces

$$|U_i|\leqslant U_i^0 \tag{1.4}$$

We will make certain simplifying assumptions concerning the kinetic energy and the generalized forces Q_i. It is assumed that the matrix $A(q)$ from (1.2) can be represented in the form

†*Prikl. Mat. Mekh.* Vol. 62, No. 1, pp. 121–128, 1998.

$$A(q) = I + \tilde{A}(q), \quad I = \mathrm{diag}(I_1, \ldots, I_n), \quad I_i = \mathrm{const} > 0 \tag{1.5}$$

where $\tilde{A}(q)$ is a symmetric matrix such that, for any n-dimensional vector z, the inequality

$$|\tilde{A}(q)z| \leq \mu|z|, \quad \mu > 0, \quad \forall q \in D \tag{1.6}$$

is satisfied. Here μ is a sufficiently small parameter, possible values of which are indicated below.

Furthermore, we assume that

$$|\partial A_{jk} / \partial q_i| \leq c, \quad c = \mathrm{const} > 0 \tag{1.7}$$

and that the generalized forces Q_i can be represented in the form

$$Q_i = G_i + F_i \tag{1.8}$$

Here $G_i(q, \dot{q}, t)$ are restricted forces, the magnitudes of which do not exceed the permissible values of the control forces, that is

$$|G_i| \leq G_i^0, \quad G_i^0 < U_i^0 \tag{1.9}$$

where G_i^0 are specified constants. Note that, if the inequality $G_i^0 > U_i^0$, which is the inverse of (1.9), holds for certain i then the system can be uncontrollable.

The forces, which are sufficiently small at low velocities and satisfy the constraints

$$|F_i| \leq a|\dot{q}| + b|\dot{q}|^2 \tag{1.10}$$

where a and b are certain positive constants, are denoted by $F_i(q, \dot{q}, t)$ in (1.8). The exact form of the functions $G_i(q, \dot{q}, t)$, $F_i(q, \dot{q}, t)$ in (1.8) may be unknown.

We now formulate the control problem.

Problem 1. It is required to determine the control functions $U_i(q_i, \dot{q}_i)$ which satisfy constraints (1.4) and ensure that system (1.1) is transferred from a specified initial state

$$q(0) = q^0, \quad \dot{q}(0) = \dot{q}^0, \quad q^0 \in D \tag{1.11}$$

to a specified final stationary state

$$q(\tau) = q^1, \quad \dot{q}(\tau) = 0, \quad q^1 \in D \tag{1.12}$$

The time of the control process τ is finite and is not fixed. Without loss of generality, the initial instant of time is taken to be equal to zero.

2. DECOMPOSITION OF THE SYSTEM

The method of decomposition, proposed in [1], is used to solve the problem. We substitute expression (1.2) for the kinetic energy T into (1.1) and write the equations of motion in the vector form

$$A(q)\ddot{q} = U + G + S(q, \dot{q}, t) \tag{2.1}$$

Here $U = (U_1, \ldots, U_n)$ is the vector of the controls, $G = (G_1, \ldots, G_n)$ is the vector of the restricted forces (1.9), and $S = (S_1, \ldots, S_n)$ is a vector-function with the components

$$S_i(q, \dot{q}, t) = F_i(q, \dot{q}, t) + \sum_{j,k} \left(\frac{1}{2} \frac{\partial A_{jk}}{\partial q_i} - \frac{\partial A_{ij}}{\partial q_k} \right) \dot{q}_j \dot{q}_k \tag{2.2}$$

Note that the quantities S_i vanish when $\dot{q} = 0$.

We multiply both sides of Eq. (2.1) by IA^{-1} (the matrix I has been introduced in (1.5)) and obtain

$$I_i\ddot{q}_i = U_i + V_i \tag{2.3}$$

$$V_i = G_i + S_i - [\tilde{A}A^{-1}(U + G + S)]_i \tag{2.4}$$

System (2.3), (2.4) is equivalent to the initial equation (2.1).

We assume that the inequalities

$$|V_i| \leq \rho_i U_i^0, \quad \rho_i < 1 \tag{2.5}$$

hold, where ρ_i are certain constants. We shall treat the functions V_i in (2.3) as independent restricted perturbations. In this case, the initial non-linear system is decomposed into n linear subsystems, subjected to the perturbations, where each subsystem has a single degree of freedom. To solve Problem 1, it is therefore sufficient to solve the n simpler control problems for the second-order subsystems (2.3).

The control law for each of these control systems is presented in Section 3. The conditions under which inequalities (2.5) are actually satisfied are found in Section 4.

3. CONTROL OF A LINEAR SUBSYSTEM

As has been done previously in [1], we shall specify the scalar control U_i which transfers the ith subsystem (2.3) in a finite time from the arbitrary initial state $(q_i^0, \dot{q}_i^0)$ to the final state $(q_i^1, 0)$ for any permissible perturbation V_i, which satisfies (2.5) in the form of a synthesis

$$\begin{aligned} &U_i(q_i, \dot{q}_i) = U_i^0 \operatorname{sign} \psi_i(q_i, \dot{q}_i), \quad \psi_i \neq 0 \\ &U_i(q_i, \dot{q}_i) = -U_i^0 \operatorname{sign} \dot{q}_i, \quad \psi_i = 0 \\ &\psi_i(q_i, \dot{q}_i) = q_i^1 - q_i - \dot{q}_i|\dot{q}_i|/(2X_i) \end{aligned} \tag{3.1}$$

Here X_i is the positive control parameter

$$X_i = U_i^0(1 - \rho_i)/I_i \tag{3.2}$$

Note that the value of X_i is unknown up to now, since the constant ρ_i is unknown.

The above-mentioned control was obtained as the time-optimal control in a game problem in which U_i and V_i are considered as the controls of two players [7]. This control is a bang-bang control and takes its limiting permissible values of $U_i = \pm U_i^0$. The switching curve (SC) $\psi_i(q_i, \dot{q}_i) = 0$ consists of two parabolic branches which are symmetric about the point $(q_i^1, 0)$.

We will now specify the set Ω_i in the two-dimensional phase space of the ith subsystem (Fig. 1)

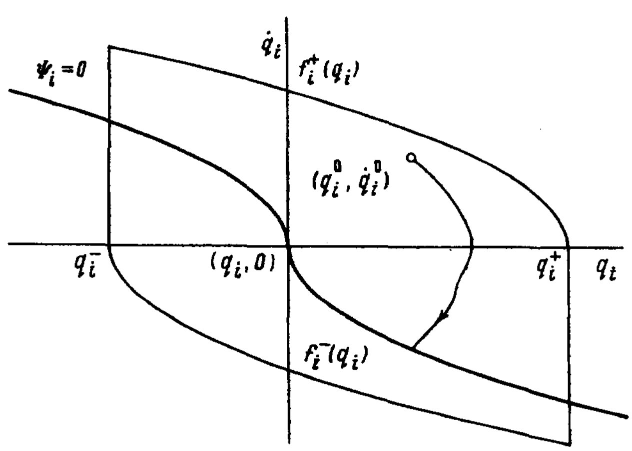

Fig. 1.

$$\Omega_i = \{(q_i, \dot q_i):\ q_i^- \le q_i \le q_i^+,\ f_i^- \le \dot q_i \le f_i^+\}$$
$$f_i^-(q_i) = -[2X_i(q_i - q_i^-)]^{1/2},\quad f_i^+(q_i) = [2X_i(q_i^+ - q_i)]^{1/2} \tag{3.3}$$

We will describe the nature of the motion of subsystem (2.3) in the case when the control is specified in the form (3.1), (3.2) and the initial point $(q_i^0, \dot q_i^0)$ lies in Ω

$$(q_i^0, \dot q_i^0) \in \Omega_i \tag{3.4}$$

The control process is divided into two main stages. In the first stage, the motion is performed with a constant control until a phase point of the subsystem reaches the SC. In this case, by (2.3), (2.5), (3.1) and (3.2), we have (to fix our ideas, we assume that $\psi_i(q_i^0, \dot q_i^0) < 0$)

$$\ddot q_i \le -X_i \tag{3.5}$$

It follows from (3.5) that the quantity $\dot q_i$ decreases and, by virtue of (3.3), (3.4) and (3.5), the inequalities

$$\frac{d\dot q_i}{dq_i} \le -\frac{X_i}{f_i^+(q_i)} = \frac{df_i^+(q_i)}{dq_i},\quad \dot q_i > 0;\quad \frac{d\dot q_i}{dq_i} > 0,\quad \dot q_i < 0$$

are satisfied.

Therefore, for any perturbations, the phase trajectory of the subsystem under consideration does not go out beyond the limits of the domain Ω_i and reaches one of the branches of the SC. This fact is proved in a similar way when $\psi_i(q_i^0, \dot q_i^0) > 0$).

On reaching the SC, the phase point continues to move along it into the terminal state. The parabolic branches of the SC coincide with the phase trajectories of subsystem (2.3) in the case of a control U_i selected in accordance with (3.1) and (3.2) and when $V_i = -\rho_i U_i$. If, however, $V_i \ne -\rho_i U_i$, the motion occurs along a parabolic segment in the same way but in a sliding mode. In this case, the control U_i takes the values $\pm U_i^0$ with infinitely frequent changes of sign so that "on average" $\ddot q_i = X_i$ or $\ddot q_i = -X_i$ for the corresponding branches of the switching curve.

Hence, if conditions (3.3) and (3.4) are satisfied for all the subsystems (2.3) at the initial instant of time, then their phase trajectories as a whole lie in the corresponding domains Ω_i. In this case, constraints (1.3) are satisfied and the inequalities

$$|\dot q_i| \le (2d_iX_i)^{1/2},\quad d_i = q_i^+ - q_i^- \tag{3.6}$$

also hold. A certain possible phase trajectory of subsystem (2.3) is shown in Fig. 1. The direction of increasing time t is indicated by the arrows.

It has been shown [1] that the time for the motion of the ith subsystem (2.3) is a maximum in the case of the "worst" perturbation $V_i = -\rho_i U_i$ and is equal to

$$\tau_i^*(q_i^0, \dot q_i^0) = X_i^{-1}\{2[(\dot q_i^0)^2/2 - X_i(q_i^0 - q_i^1)\gamma_i]^{1/2} - \dot q_i^0\gamma_i\}$$
$$\gamma_i = \operatorname{sign}\psi_i(q_i^0, \dot q_i^0),\quad \psi_i \ne 0;\quad \gamma_i = \pm 1,\quad \psi_i = 0 \tag{3.7}$$

Since the time τ required to bring system (1.1) to the terminal state (1.12) is defined as the greatest of the control times for each of the subsystems (2.3), we obtain the estimate

$$\tau \le \tau^* = \max_i(\tau_i^*) \tag{3.8}$$

4. FINDING THE PERMISSIBLE PARAMETERS X_i

Control (3.1) can only be used when inequalities (2.5) are satisfied throughout the whole of the control process. We shall now find those control parameters X_i for which the above-mentioned relations are, in fact, satisfied.

We will first estimate the moduli of the quantities V_i. When $\mu < I_{\min}$, using relations (1.4)–(1.6), we obtain

$$|V_i| \leq G_i^0 + \left(1 + \frac{\mu n^{1/2}}{I_{\min} - \mu}\right) S^0 + \frac{\mu}{I_{\min} - \mu}\left[\sum_j (U_j^0 + G_j^0)^2\right]^{1/2} \tag{4.1}$$

Here, $I_{\min}$ is the least of the quantities I_i and S^0 is a constant, which restricts the absolute values of the functions $S_i(q, \dot{q}, t)$ from (2.2) for control (3.1) with parameters X_i. Subject to constraints (1.7), (1.10) and (3.6), we have

$$S^0(X) = a\left(2\sum_j d_j X_j\right)^{1/2} + 2b\sum_j d_j X_j + 3c\left[\sum_j (d_j X_j)^{1/2}\right]^2 \tag{4.2}$$

$$X = (X_1, \ldots, X_n)$$

In inequalities (2.5), we express ρ_i in terms of the control parameters X_i using (3.2) and, instead of the quantities $|V_i|$, we substitute their estimates from (4.1). We obtain

$$I_i X_i + \left(1 + \frac{\mu n^{1/2}}{I_{\min} - \mu}\right) S^0(X) \leq U_i^0 - G_i^0 - \frac{\mu}{I_{\min} - \mu}\left[\sum_j (U_j^0 + G_j^0)^2\right]^{1/2} \tag{4.3}$$

If the parameter μ is sufficiently small, such that the condition

$$\mu < \frac{\min_i (U_i^0 - G_i^0) I_{\min}}{\min_i (U_i^0 - G_i^0) + \left[\sum_j (U_j^0 + G_j^0)^2\right]^{1/2}} \tag{4.4}$$

is satisfied, the expressions on the right-hand sides of inequalities (4.3) are positive. Since $S^0(X) \to 0$ when $X_i \to 0$, positive values of X_i can always be found, for which inequalities (4.3) and, consequently, inequalities (2.5) are satisfied.

We will now sum up the results we have obtained in the form of a theorem.

Theorem 1. Suppose that condition (4.4) is satisfied. Then, the synthesis of the control $U_i(q_i, \dot{q}_i)$, which solves Problem 1, is specified by relations (3.1) in which the parameters X_i must be chosen in such a way that inequalities (4.3) are satisfied. This control transfers system (1.1) from the initial state (1.11) to the specified terminal state (1.12), if, at the initial instant of time, the quantities $\dot{q}_i^0$ satisfy the constraints $f_i^-(q_i^0) \leq \dot{q}_i^0 \leq f_i^*(q_i^0)$. In this case, the motion of the system lies in the domain D from (1.3) and the time of the control process τ does not exceed $\tau*$, which is determined by expressions (3.7) and (3.8).

We will now describe a method of selecting the permissible values of X_i. We shall seek these values in the form

$$X_i = Y^2 d_i \tag{4.5}$$

where the magnitude of Y is still unknown. We substitute (4.5) into inequalities (4.3) and reduce them to the form

$$Y^2 + 2g_i Y \leq h_i \tag{4.6}$$

where g_i, h_i are positive coefficients, the explicit form of which immediately follows from (4.2) and (4.3). The solution of the system of inequalities (4.6) can be written in the form

$$Y \leq \min_i [(g_i^2 + h_i)^{1/2} - g_i]$$

Selecting the maximum value of Y which satisfies the inequality obtained, we calculate the control parameters X_i using formulae (4.5).

5. APPLICATIONS TO ROBOT CONTROL PROBLEMS

We will now consider a manipulating robot consisting of n absolutely rigid links joined to one another by means of cylindrical or prismatic hinges. The position of the links of the robot in space is characterized by their relative angles of rotation (in the case of cylindrical hinges) or relative displacements (in the case of prismatic hinges). We will take these angles and displacements as the generalized coordinates $q = (q_1, \ldots, q_n)$. If the equations of motion of the robot are represented in the form (1.1) and (1.2), the moments of the forces with respect to the axes of the cylindrical hinges and the forces acting in the direction of the displacements in prismatic hinges will play the role of generalized forces. In this case, U_i are the control forces or the moments of the forces produced by the electromechanical drives of the robot and Q_i are all the remaining external and internal forces and moments which arise as the result of the action of gravitational forces, friction, various perturbations, etc. We shall next assume that the forces Q_i can be represented in the form (1.8)–(1.10).

The kinetic energy of the robot T is made up of the kinetic energy of the motion of the links $T^1(q, \dot{q})$ and the kinetic energy of the motion of the rotors of the electric motors $T^2(q, \dot{q}, N)$. Here, $N = (N_1, \ldots, N_n)$ are the gear ratios of the reduction gears, which are treated as parameters. We shall assume that $N_i \geqslant 1$ and neglect the inertia of the moving parts of the reduction gears. According to König's theorem, the kinetic energy of the ith rotor is equal to the sum of the kinetic energy which a point mass with a mass equal to the mass of the rotor, located at its centre of inertia, would have and the kinetic energy of rotation of the rotor, that is

$$T_i^2(q,\dot{q},N_i) = T_i^v(q,\dot{q}) + T_i^\omega(q,\dot{q},N_i)$$

Suppose that J_i, J_i' are the moments of inertia of the ith rotor about its axis of rotation and an axis passing through the centre of inertia perpendicular to the axis of rotation. Then, if the angular velocity vector of the stator of the ith electric motor has a projection on the axis of rotation of the rotor equal to ω_i and a perpendicular component equal to ω_i', we have

$$T_i^\omega(q,\dot{q},N_i) = \tfrac{1}{2}[J_i(N_i\dot{q}_i + \omega_i)^2 + J_i'\omega_i'^2]$$

The angular velocities ω_i, ω_i' are linear combinations of the generalized velocities $\dot{q}_1, \ldots, \dot{q}_n$ with coefficients which depend on q. The kinetic energy of the robot can therefore be represented in the form

$$T = \frac{1}{2}\sum_j J_j(N_j\dot{q}_j)^2 + \frac{1}{2}N_{\max}(B\dot{q},\dot{q}) \tag{5.1}$$

where $B(q, N)$ is a bounded matrix such that the inequality

$$|B(q,N)z| \leqslant \lambda|z|, \quad \lambda = \text{const} \tag{5.2}$$

is satisfied in the case of an arbitrary vector z.

The largest and smallest of the gear rations $N_1, \ldots, N_n$ are henceforth denoted by $N_{\max}$ and $N_{\min}$.

We substitute (5.1) into the Lagrange equations in the form of (1.1) and obtain

$$N_i^2 J_i\ddot{q}_i + N_{\max}[B(q,N)\ddot{q}]_i = U_i + G_i + S_i(q,\dot{q},t,N) \tag{5.3}$$

We divide the ith equation of (5.3) by N_i and make the change of variables

$$p_i = N_i q_i \tag{5.4}$$

As a result, we obtain

$$J_i\ddot{p}_i + N_{\max}N_i^{-1}\sum_j B_{ij}N_j^{-1}\ddot{p}_j = N_i^{-1}(U_i + G_i + S_i) \tag{5.5}$$

Allowing for the fact that $N_i^{-1}U_i = M_i$, where M_i is the electromagnetic moment produced by the electric motor, we reduce system (5.5) to the form

$$(J + \tilde{B})\ddot{p} = M + G^* + S^* \tag{5.6}$$

Here

$$J = \operatorname{diag}(J_1,\dots,J_n),\quad \tilde{B} = N_{\max} HBH,\quad M = (M_1,\dots,M_n)$$
$$G^* = HG,\quad S^* = HS,\quad H = \operatorname{diag}(N_1^{-1},\dots,N_n^{-1}) \tag{5.7}$$

Consequently, when account is taken of the change of variables (5.4) and the notation (5.7), the equations of motion can be represented in the form of (1.5) and (2.1), and, by (5.2) and (5.7), we have the inequality

$$|\tilde{B}z| \leqslant \mu^* |z|,\quad \mu^* = N_{\max} N_{\min}^{-2} \lambda \tag{5.8}$$

which is analogous to constraint (1.6). The initial and final conditions can be represented in the form (1.11) and (1.12).

We will now consider different ways of formulating control problems.

1. Suppose that the constraints

$$|M_i| \leqslant M_i^0 \tag{5.9}$$

are imposed on the control moments of the forces M_i produced by the electric motors. In this case, the results obtained in the preceding sections, which have been summarized in Theorem 1, can be used to construct the control. Inequality (4.4), rewritten in the notation of system (5.6), defines the permissible values of the parameter μ^*. On substituting its value from (5.8) into this inequality instead of μ^* we obtain a constraint on the possible values of the gear ratios of the reduction gears

$$\frac{N_{\min}^2}{N_{\max}} > \frac{\lambda}{J_{\min}} \left(1 + \frac{\left[\sum_j (M_j^0 + G_j^{*0})^2 \right]^{1/2}}{\min_i (M_i^0 - G_i^{*0})} \right) \tag{5.10}$$

Here G_i^{*0} is a constant which bounds the absolute values of the functions G_i^*, and $J_{\min}$ is the least of the moments of inertia of the rotors $J_1, \dots, J_n$.

2. Suppose that the voltages applied to the windings of the electric motors play the role of controls. We augment the equations of motion (5.6) with the balance equations for the voltages in the rotor circuits and relations associating the moments M_i with the currents

$$L_i \frac{dj_i}{dt} + R_i j_i + k_i^E \dot{p}_i = u_i,\quad M_i = k_i^M j_i \tag{5.11}$$

Here L_i is the coefficient of inductance, R_i is the electrical resistance, k_i^E, k_i^M are constant coefficients, and u_i is the voltage in the rotor circuit of the ith motor. The first term in the first equation of (5.11) is usually small compared with the remaining terms and the expression

$$M_i = k_i^M R_i^{-1} (u_i - k_i^E \dot{p}_i)$$

is therefore obtained from (5.11) and, when this is substituted into (5.6), we obtain

$$(J + \mu^* \tilde{B})\ddot{p} = U^* + G^* + S^{**} \tag{5.12}$$
$$S^{**} = S^* - \Lambda \dot{p},\quad \Lambda = \operatorname{diag}(k_1^M k_1^E R_1^{-1},\dots,k_n^M k_n^E R_n^{-1})$$
$$U^* = (k_1^M R_1^{-1} u_1,\dots,k_n^M R_n^{-1} u_n)$$

Suppose that the constraints

$$|u_i| \leqslant u_i^0 \tag{5.13}$$

are imposed on the control voltages.

The constraints (5.13) are transformed into constraints on the components of the vector U^* from (5.12)

$$|U_i^*| \leq U_i^{*0} = k_i^M R_i^{-1} u_i^0 \tag{5.14}$$

The equations of motion (5.12) are again reduced to the form (1.5) and (2.1).

Inequalities (5.14) are of the same form as relations (1.4). It is obvious that in this case we can use the method of control considered. By Theorem 1, we obtain a constraint which is analogous to (5.10)

$$\frac{N_{\min}^2}{N_{\max}} > \frac{\lambda}{J_{\min}}\left(1 + \frac{\left[\sum_j (k_j^M R_j^{-1} u_j^0 + G_j^{*0})^2\right]^{1/2}}{\min_i (k_i^M R_i^{-1} u_i^0 - G_i^{*0})}\right) \tag{5.15}$$

So, if the year ratios of the drives and the parameters of the robot are such that inequalities (5.10) and (5.15) are satisfied, it is possible to construct a control which transfers the system under consideration from an initial state to a specified state in a finite time. The control takes account of the existence of perturbations and structural constraints.

This research was supported financially by the Russian Foundation for Basic Research (96-01-01137) and the Program for the Sponsorship of Leading Scientific Schools (96-15-96236).

REFERENCES

1. CHERNOUS'KO, F. L., Decomposition and suboptimal control in dynamical systems. *Prikl. Mat. Mekh.*, 1990, **54**, (6), 883–893.
2. CHERNOUS'KO, F. L., Control synthesis in a non-linear dynamical system. *Prikl. Mat. Mekh.*, 1992, **56**, (2), 179–191.
3. CHERNOUS'KO, F. L., The decomposition of controlled dynamic systems. In *Advances in Nonlinear Dynamics and Control*, (ed. by Kurzhanski, A.B.). Birkhäuser, Boston, 1993, 1–40.
4. CHERNOUS'KO, F. L., Decomposition and synthesis of a control in non-linear dynamical systems. *Tr. Mat. Inst. Ross. Akad. Nauk*, 1995, **211**, 457–472.
5. PYATNITSKII, Ye. S., Synthesis of the control of manipulating robots using the principle of decomposition. *Izv. Akad. Nauk SSSR. Tekhn. Kibernetika*, 1987, 3, 92–99.
6. PYATNITSKII, Ye. S., The principle of decomposition in the control of mechanical systems. *Dokl. Akad Nauk SSSR*, 1988, **300**, (2), 300–303.
7. KRASOVSKII, N. N., *Game Problems of the Encounter of Motions*. Nauka, Moscow, 1970.

Translated by E.L.S.